KEPLERS LAWS AS APPLIED TO SATELLITES

ECE 514E – RADAR & SATELLITE ENGINEERING

Tuesday, September 24, 2024

OPPORTUNITIES IN SATELLITE & RADAR ENGINEERING

About us

KSA is mandated to promote, coordinate and regulate space related activities in the country. This will be achieved through promotion of research and innovations in space science, technology and respective applications as well as enhancing the regulatory framework. It will also spur Kenya's competitiveness and positioning in playing a critical role in the regional and global space agenda

Mission

To coordinate, nurture and develop Kenya's Space sector towards enhanced utilization of Space opportunities.

Vision

Effective utilization of Space Economy for national sustainable development.

Regulation

1. National Airspace
2. Outer space
3. Deep space

Mandate

Promote, coordinate and regulate space related activities in Kenya.

Core Values

- Excellence
- Professionalism
- Integrity
- Commitment

History

1964

Kenya & Italy collaborate to establish a Satellite launching and tracking base in Malindi

Sounding rocket (rocketsonde)
An instrument-carrying rocket designed to take measurements and perform scientific experiments during its sub-orbital flight.

1967 - 1989

Over 20 sounding rockets & 9 rockets launched from the Malindi Station

Space Economy
Enables new applications, in sectors such as meteorology, energy, telecommunications, insurance, transport, maritime, aviation and urban development.

2009

National Space Secretariat was established

2017

Kenya Space Agency established as successor to the National Space Secretariate

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[About | Kenya Space Agency (ksa.go.ke)](https://ksa.go.ke)

OPPORTUNITIES IN SATELLITE & RADAR ENGINEERING

Mapping for Sustainable Development | Kasarani, Nairobi, Kenya | +254 723 786161 | rcmrd@rcmrd.org

REGIONAL CENTRE FOR MAPPING OF RESOURCES FOR DEVELOPMENT

ABOUT US | PROJECTS | TRAINING | APP & DATA | MEDIA | VACANCIES | TENDERS

The Regional Centre for Mapping of Resources for Development (RCMRD) was established in Nairobi – Kenya in 1975 under the auspices of the United Nations Economic Commission for Africa (UNECA) and the then Organization of African Unity (OAU), today African Union (AU). RCMRD is an inter-governmental organization and currently has 20 Contracting Member States in the Eastern and Southern Africa Regions; Botswana, Burundi, Comoros, Eswatini, Ethiopia, Kenya, Lesotho, Malawi, Mauritius, Namibia, Rwanda, Seychelles, Somali, South Africa, South Sudan, Sudan, Tanzania, Uganda, Zambia and Zimbabwe.

Our Vision

To be a Premier Centre of Excellence in the provision of geo-information and allied technologies for sustainable development in the member States and other stakeholders.

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NATURAL & MAN-MADE SATELLITES

1. There are two different types of satellites, i.e
   a) **Natural satellites:** Planets and their moons
   b) **Artificial satellites:** Objects that are launched into space and orbits around a body in space.

2. The invention of man-made satellites used the wide pool of knowledge accumulated over five centuries from observing natural satellites.

JOHANNES KEPLER

- **Johannes Kepler (December 1571 –November 1630)** was a German astronomer, mathematician, astrologer, natural philosopher.

- He is a key figure in the 17th-century scientific revolution, best known for his laws of planetary motion.

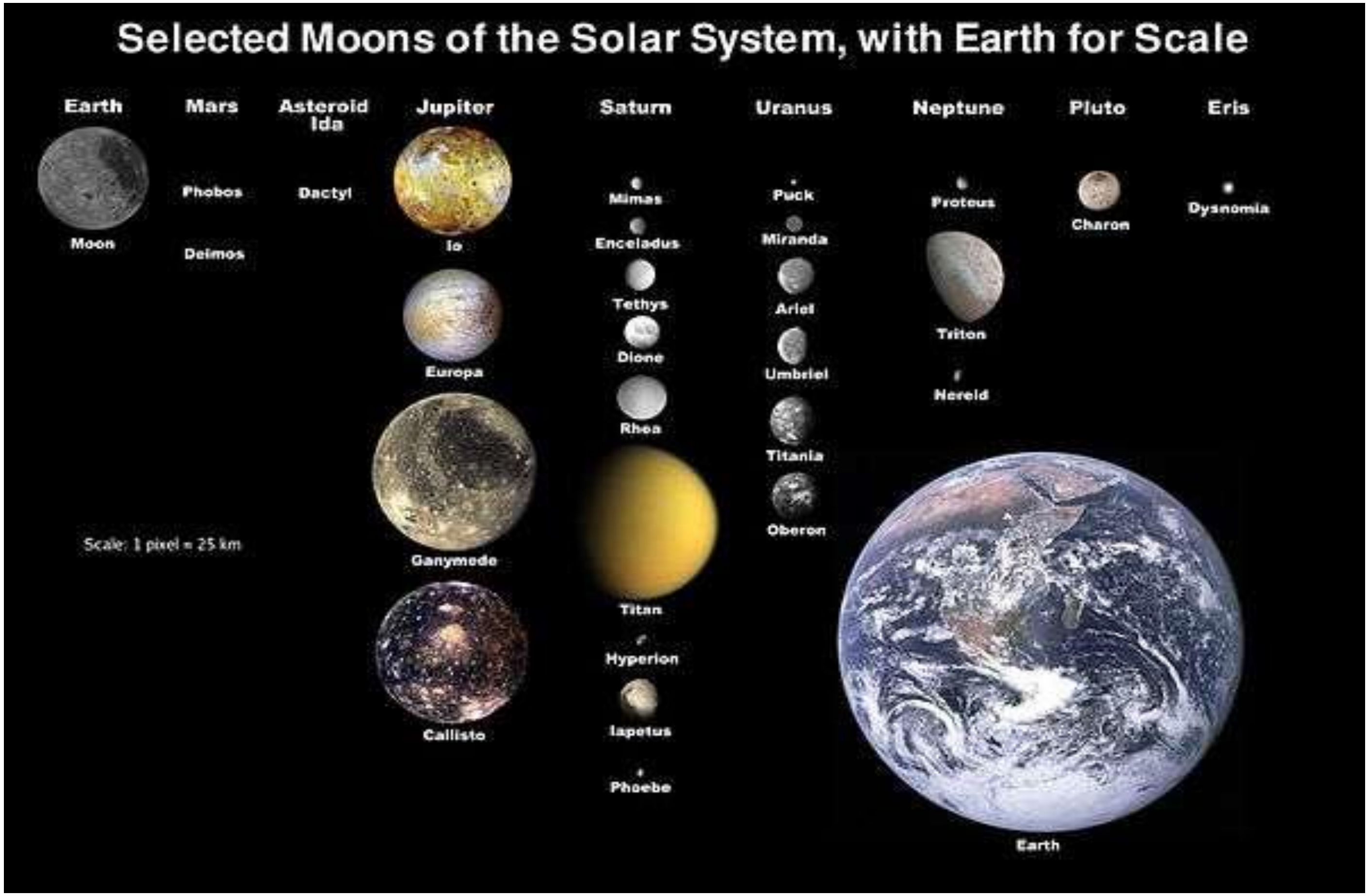
HOW KEPLER DEVELOPED HIS LAWS

- While **Nicolaus Copernicus (1473 – 1543)** observed that the planets revolve around the Sun, it was **Johannes Kepler (1571 – 1660)** who correctly defined their orbits.

- At the age of 27, Kepler became the assistant of a wealthy astronomer, **Tycho Brahe**, who asked him to define the orbit of Mars.

- Brahe had collected a lifetime of astronomical observations, which, on his death, passed to Kepler.
- Using these observations, Kepler found that the orbits of the planets followed three laws.

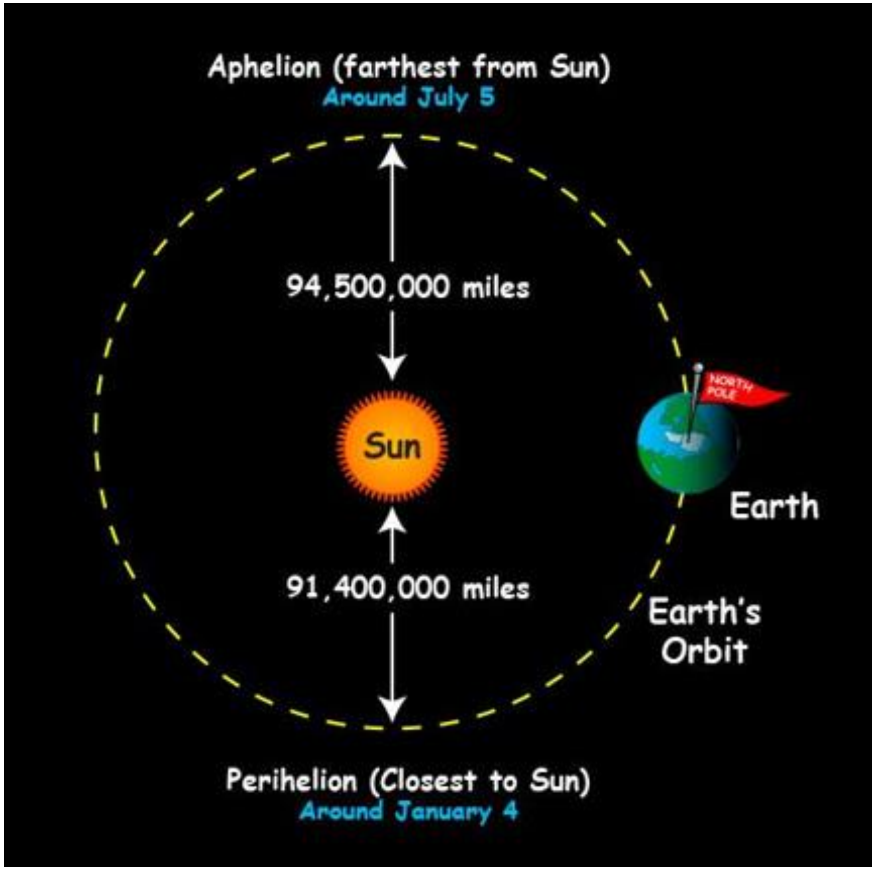
NATURAL SATELITES



1. Solar System has **178 known natural satellites** which orbit within 6 planetary satellite systems.
2. In addition, several other objects are known to have satellites, including the three dwarf planets: Pluto, Haumea, and Eris.

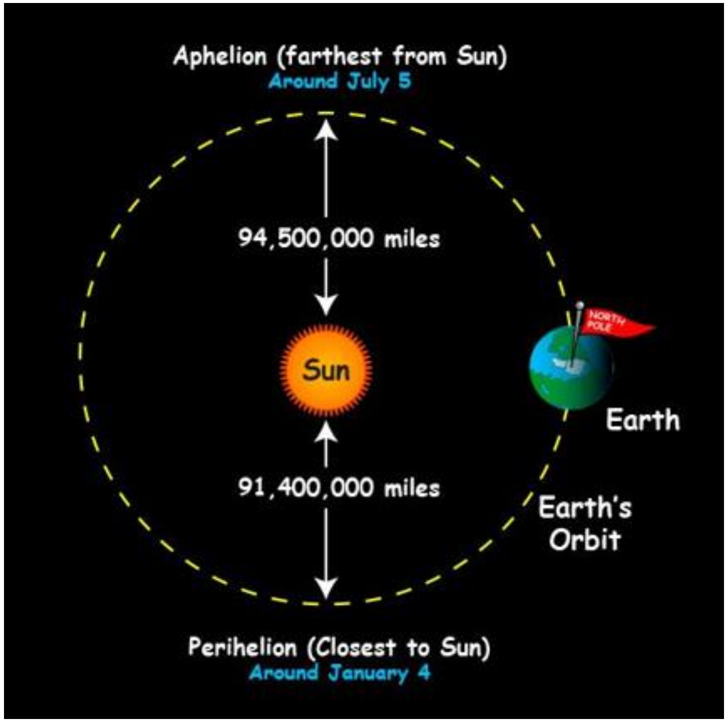
What Day is the Earth at perihelion with the Sun?

1. The sun is closest to the sun on January 4.
2. This milestone, called **perihelion** happens near the start of the Gregorian calendar year observed by much of the world.



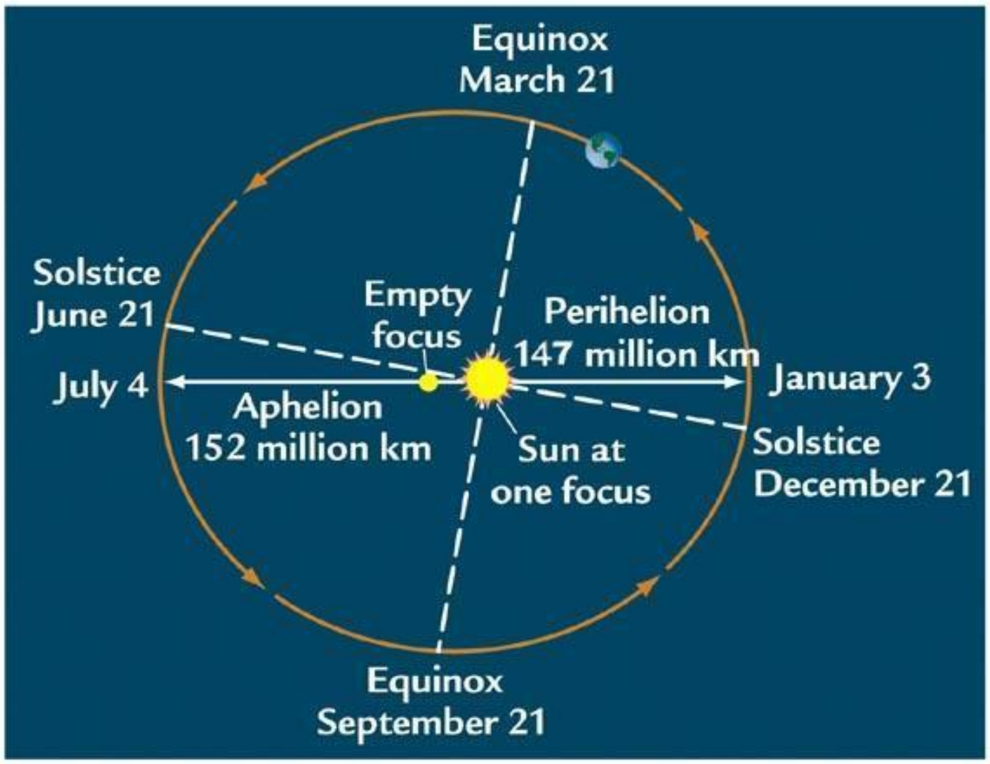
What Day is the Earth at aphelion with the Sun?

- Earth is at its **aphelion** in early July.
- Then, it is about 4,800,000 km farther from the Sun than when at its perihelion in early January.



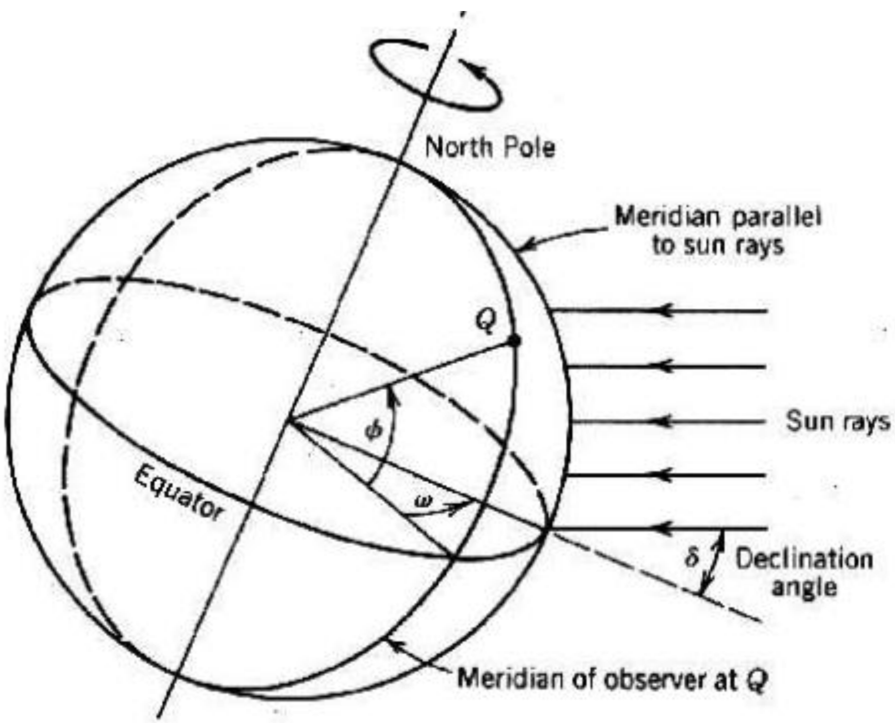
What is Equinox?

- **Equinox** - The time or date (twice each year) at which the sun crosses the equator, when day and night are of approximately equal length (about 22 September and 20 March)



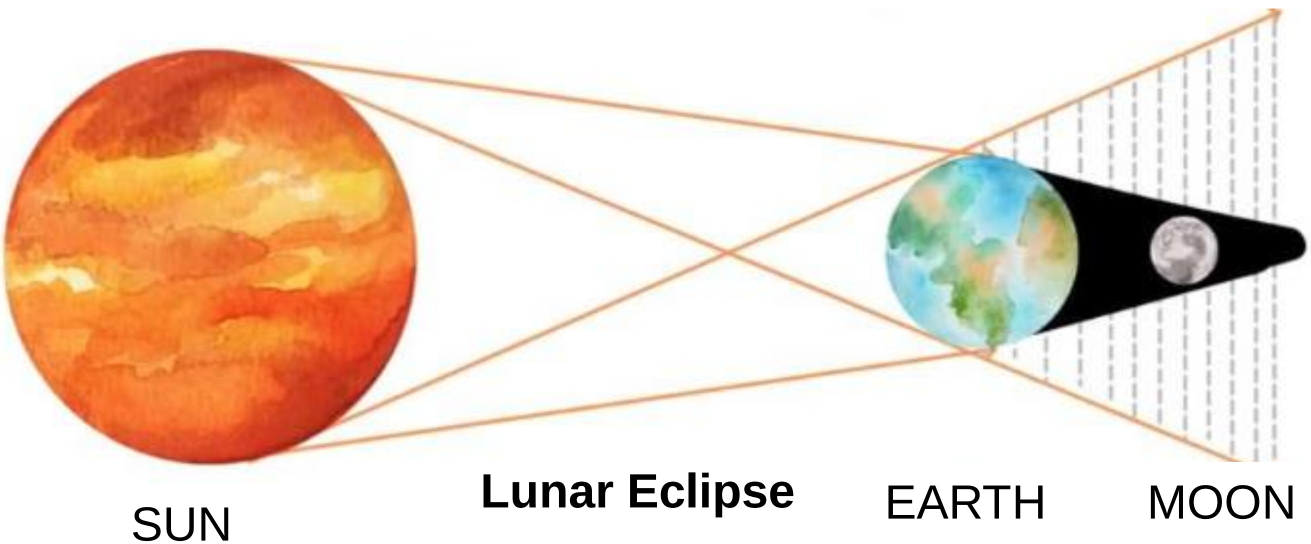
What is Solstice?

1. Solstice - the time or date (twice each year) at which the sun reaches its maximum or minimum declination.
2. Solstice is marked by the longest and shortest days (about 21 June and 22 December).



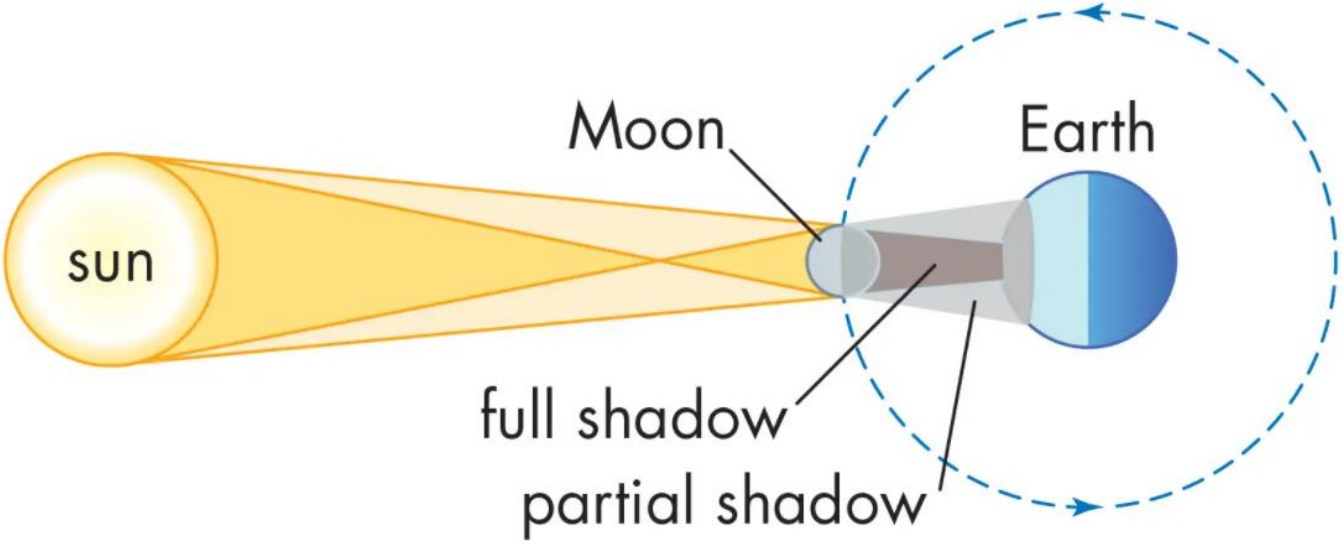
What is eclipse?

- An **eclipse** happens when one astronomical body blocks light from or to another.



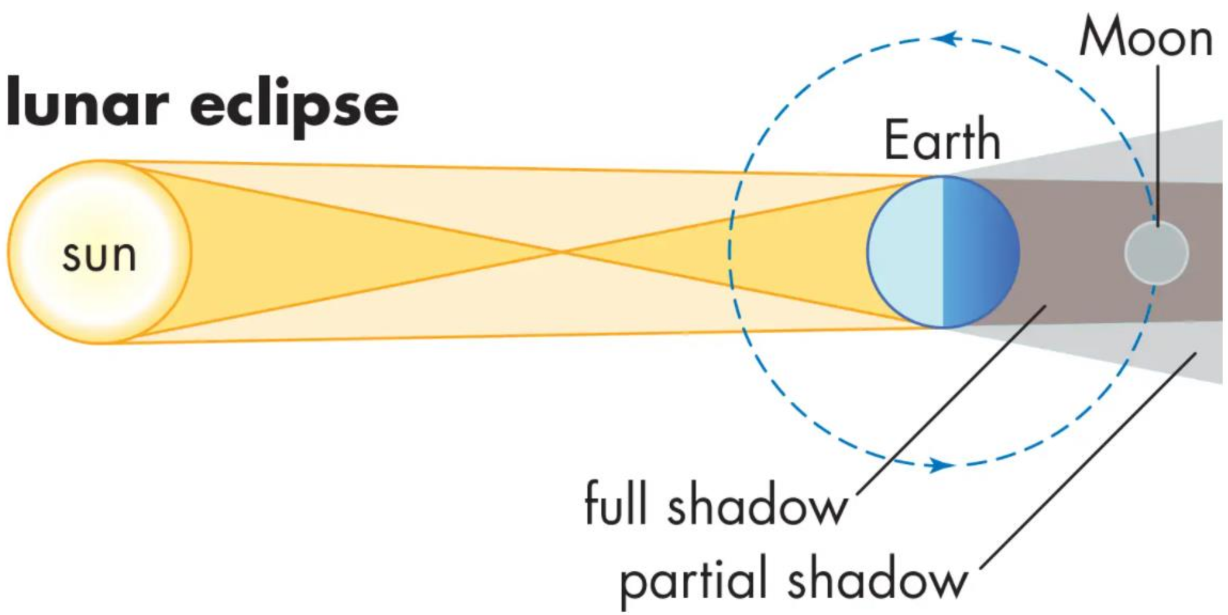
What is Solar Eclipse?

- **Solar eclipse** occurs when a portion of the Earth is engulfed in a shadow cast by the Moon which fully or partially blocks sunlight. This occurs when the Sun, Moon and Earth are aligned.



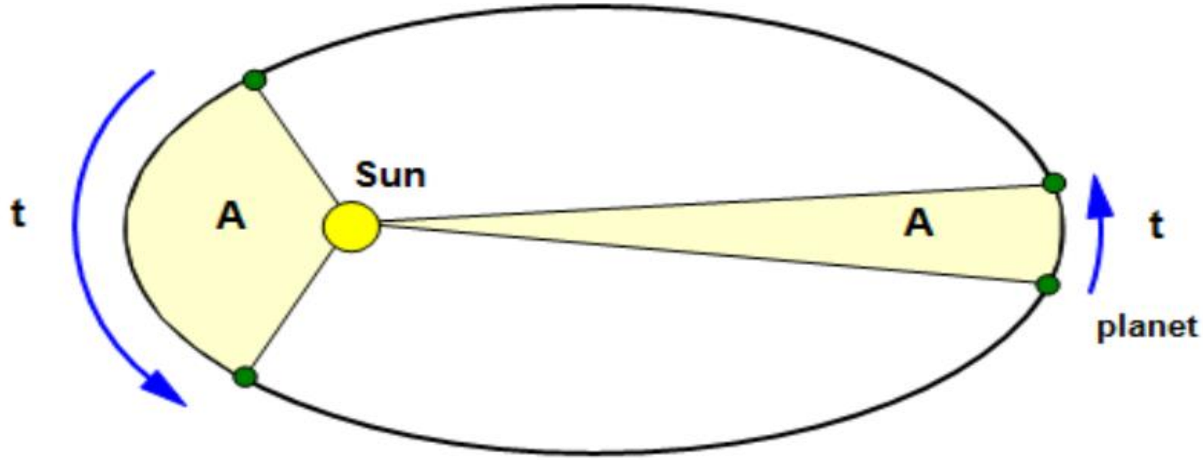
What is Lunar Eclipse?

- A **lunar eclipse** occurs when the Moon moves into the Earth's shadow.
- This occurs only when the Sun, Earth, and Moon are exactly or very closely aligned with Earth between the other two, and only on the night of a full moon.



ORIGINAL KEPLER’S LAWS

- Kepler’s laws of planetary motion are:
  1. Every planet’s orbit is an ellipse with the Sun at a focus.
  2. A line joining the Sun and a planet sweeps out equal areas in equal times.

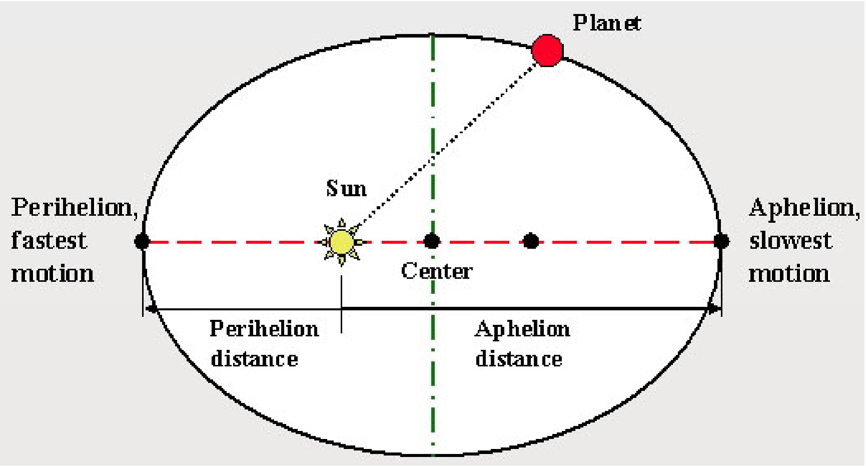


  3. The square of a planet’s orbital period is proportional to the cube of the semi-major axis of its orbit.

The third law is most often used.

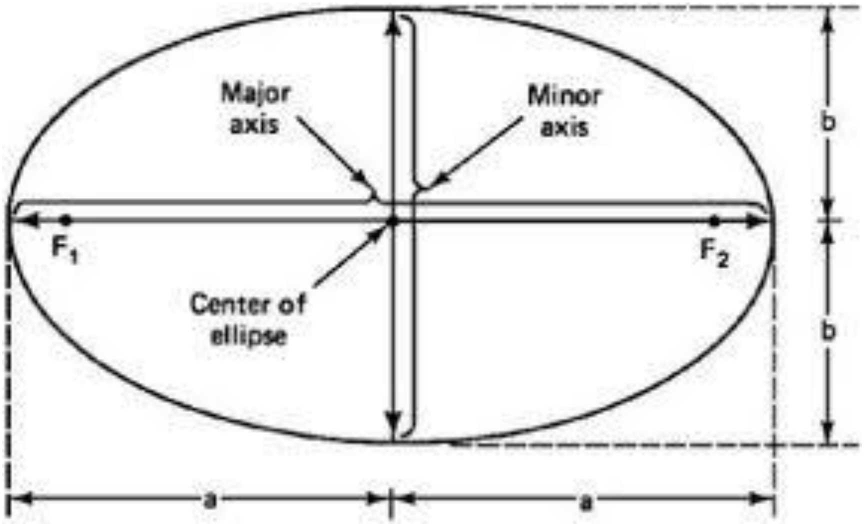
KEPLER'S LAW 1 /01

1. Whenever two bodies of mass M_1 and M_2 are in free space, such that $M_1 >> M_2$, the body with mass M_2 goes around the body of mass M_1 in an elliptical orbit.

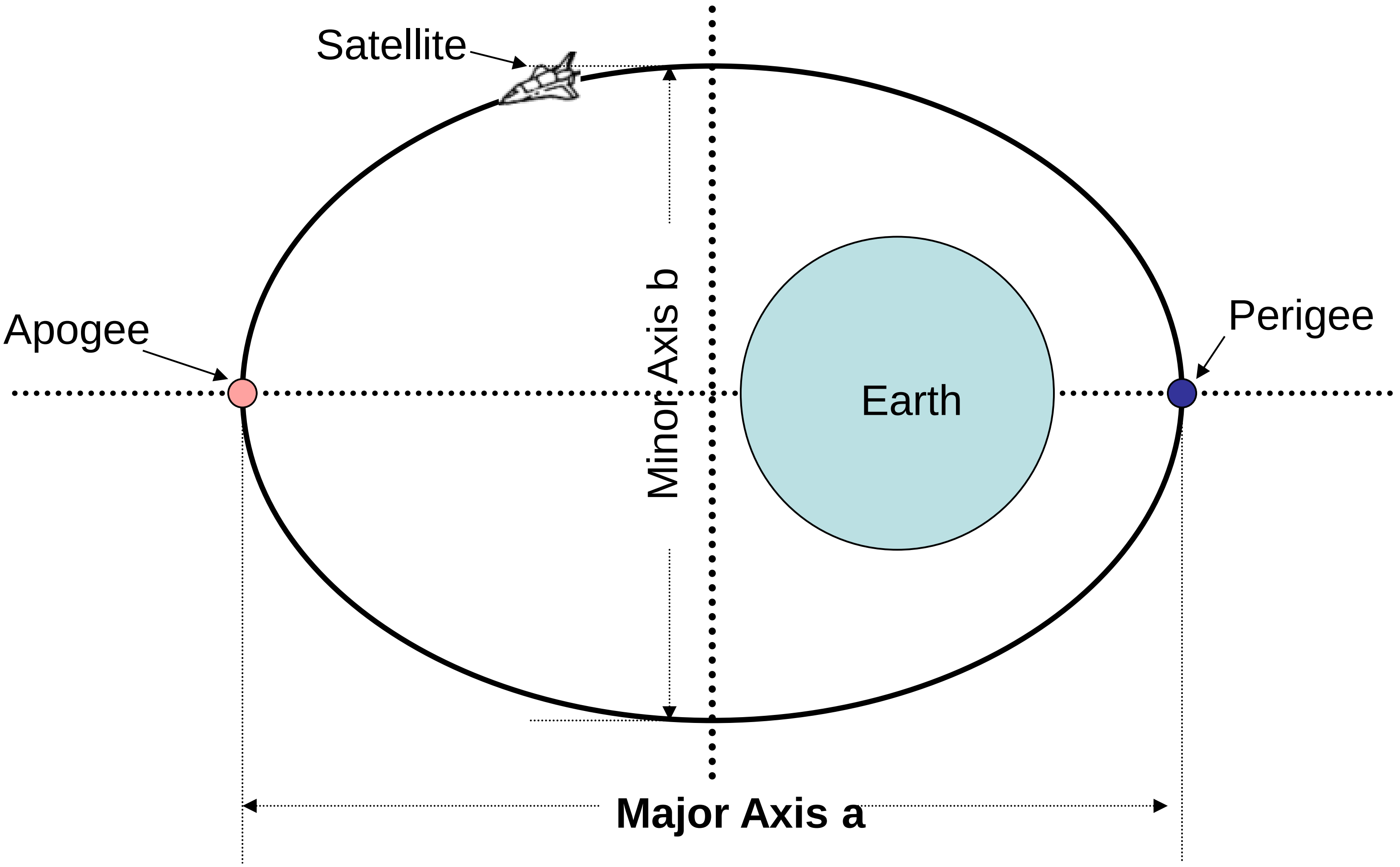


2. This happens when the centre of mass M_1 of the body lies at one of the foci of the ellipse.

3. The body M_2 is called satellite of M_1.



KEPLER'S LAW 1 /02

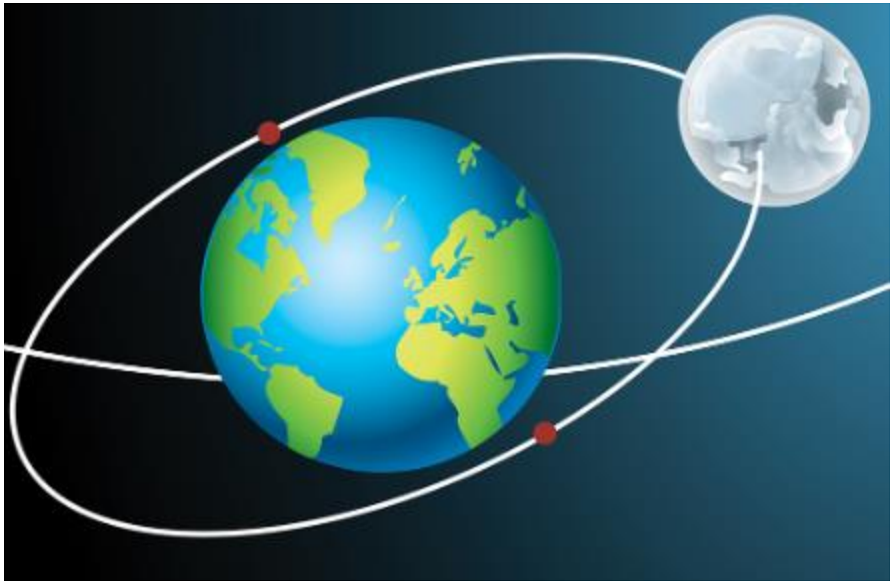


Eccentricity, e is a measure of ellipticity of an orbit.

$$e = \frac{\sqrt{a^2 - b^2}}{a} \qquad 0 \le e < 1$$

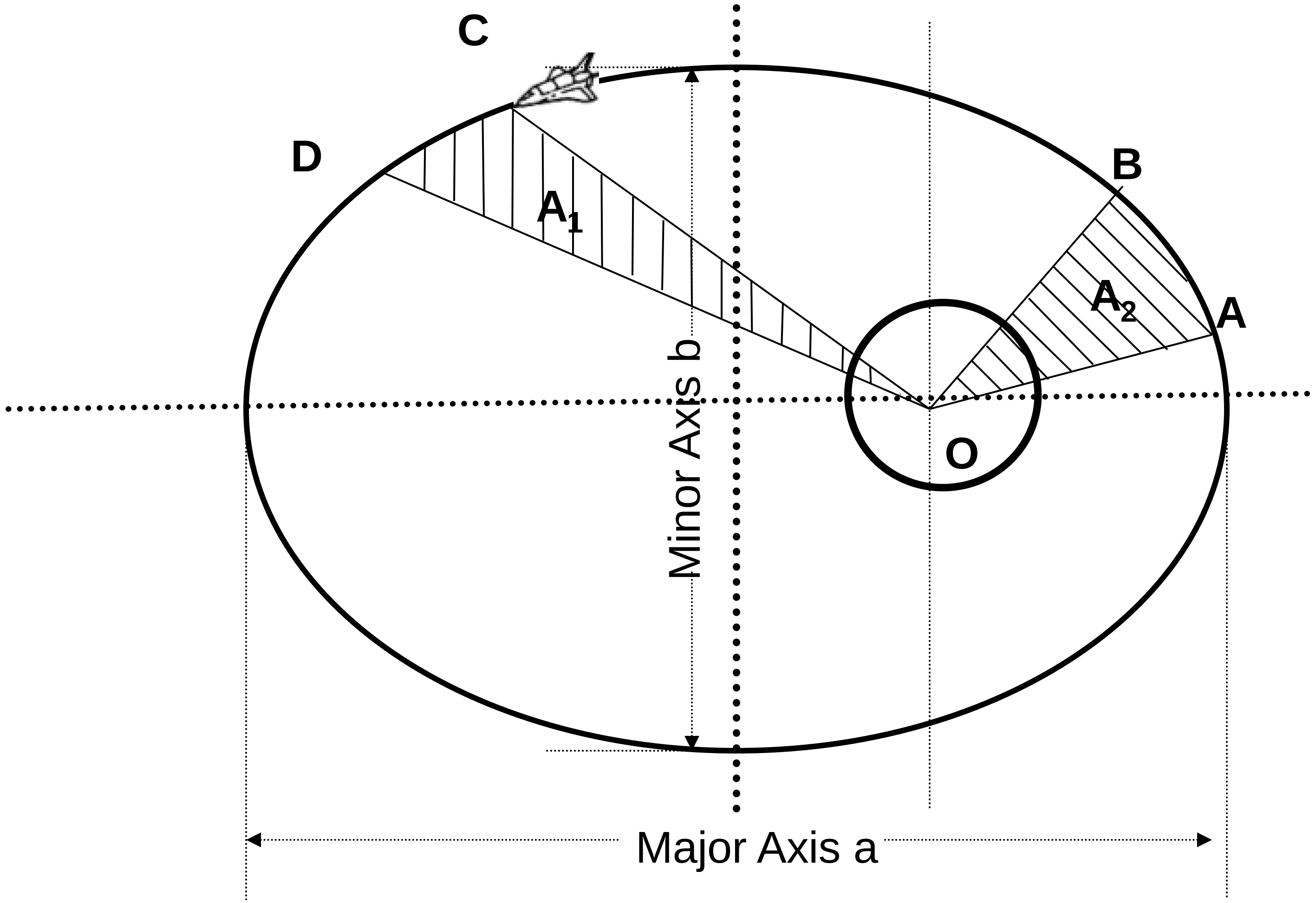
where a is major axis and b is minor axis of the elliptical path.

ORBITAL CHARACTERISTICS OF THE MOON /01



- Perigee: 362,600 km
- Apogee: 405,400 km
- Semi-major axis: 384,399 km
- Eccentricity: 0.0549
- Orbital period: 27.3 days
- Orbital period: 29.5 days
- Average orbital speed:1.022 km/s

KEPLER'S LAW 2

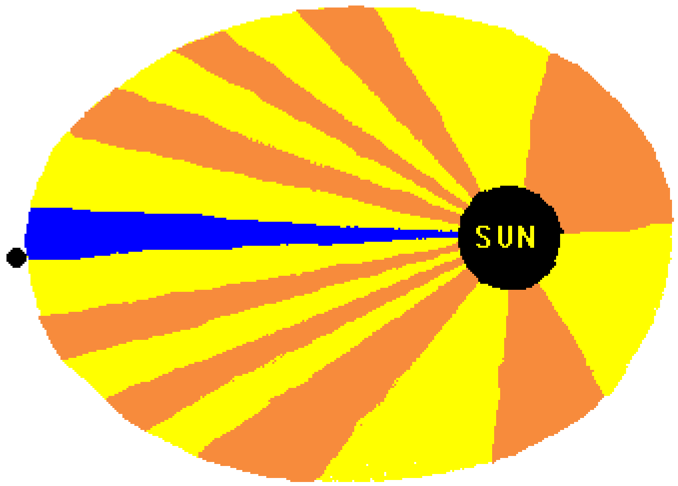


- If the satellite travels from A to B in time t and during the same time travels from C to D, then the area under OAB is equal to area OCD
- **Interpretation:** As a satellite moves away from the earth, its velocity decreases.

KEPLER’S LAW 2 (Cont..)

1. The velocity of the satellite varies depending on its position in the orbit.

2. For planets (natural satellites), the amount of radiation received is maximum at the perihelion and minimum at the aphelion.



KEPLER'S LAW 3 (1)

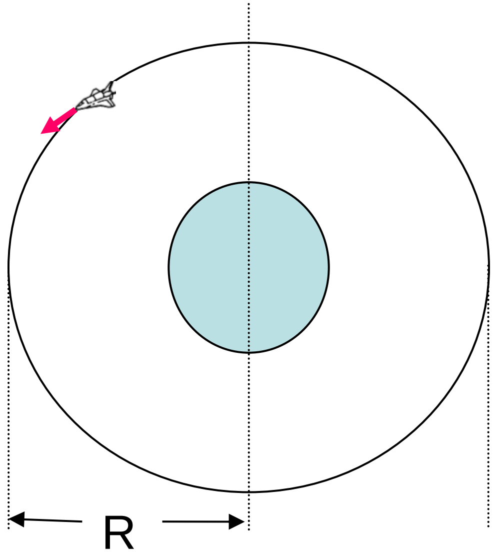
The square of the periodic time of orbit is inversely proportional to the cube of the mean distance between the two bodies or

$$R^3 = \frac{\mu}{\omega^2}$$

ω = Mean motion of the satellite in radians/sec.

μ = Earth's geocentric gravitational constant

$\mu = 3.98 \times 10^{14}\ m^3/sec^2$



EXAMPLE: ORBITAL CHARACTERISTICS OF THE MOON

Perigee:	362,600 km
Apogee:	405,400 km
Semi-major axis:	384,399 km
Eccentricity:	0.0549
Gravitational constant, μ	3.98 x 10^{14} M^3/s^2

- What is the orbital period of the moon?

$$R^3 = \frac{\mu}{\omega^2}$$

T (period) = $\frac{2\pi}{\omega}$ or $\omega = \frac{2\pi}{T}$

Substituting in equation (1), we get

$$R^3 = \frac{\mu}{(2\pi/T)^2} = \frac{\mu T^2}{(2\pi)^2}$$

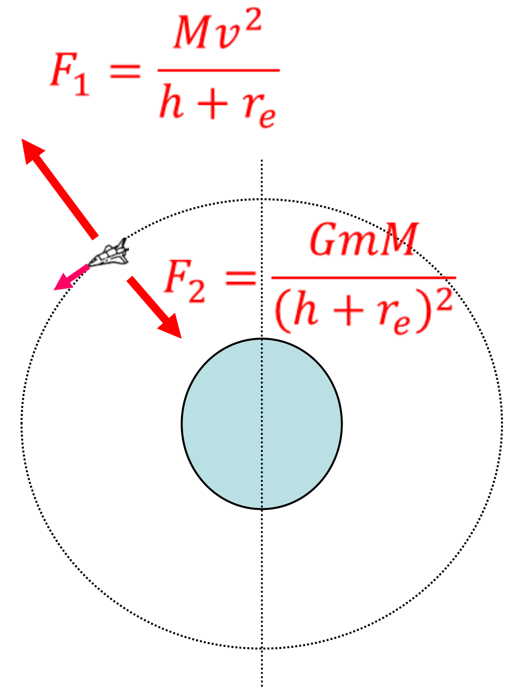
$$T^2 = \frac{R^3(2\pi)^2}{\mu}$$

$$T = 2\pi \frac{R^{3/2}}{\mu}$$

- Orbital period: 27.3 days
- Average orbital speed: 1.022 km/s

SATELLITES IN CIRCULAR ORBIT

1. Let M be the mass of the earth and m be that of the satellite.
2. If v_s is the velocity of the satellite, h is the height of the satellite and r_e is the radius of the earth then we can write:
$$v_s = \omega(r_e + h)$$
3. If F_1 is the centripetal force on the satellite, then we can write:
$$F_1 = \frac{mv_s^2}{r_e + h} = \frac{m\omega^2(r_e + h)^2}{(r_e + h)} = m\omega^2(r_e + h)$$
4. If the period of the satellite is T, then $\omega = \frac{2\pi}{T}$ and we can write:
$$F_1 = m\left(\frac{2\pi}{T}\right)^2 (r_e + h)$$
5. But the gravitational pull from the earth is given by:
$$F_2 = \frac{GMm}{(r_e + h)^2}$$
6. In equilibrium $F_1 = F_2$
$$m\left(\frac{2\pi}{T}\right)^2 (r_e + h) = \frac{GMm}{(r_e+h)^2}$$
or
$$\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{(r_e+h)^3}$$



For the earth:
$G = 6.672 \times 10^{-11}$ Newton Metre/kg^2
$M = 5.97 \times 10^{24} Kg$
Gravitational Coefficient (or Kepler's Coefficient) is:
$g_0 = GM = 3.9861 \times 10^5$ Km^3/s^2

PERIOD OF SATELLITE IN CIRCULAR ORBIT

$$\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{(r_e+h)^3}$$

This Confirms Kepler's Second law, i.e

$$R^3 = \frac{\mu}{\omega^2}$$

- Rearranging the equation and substituting $g_o = GM$ gives:

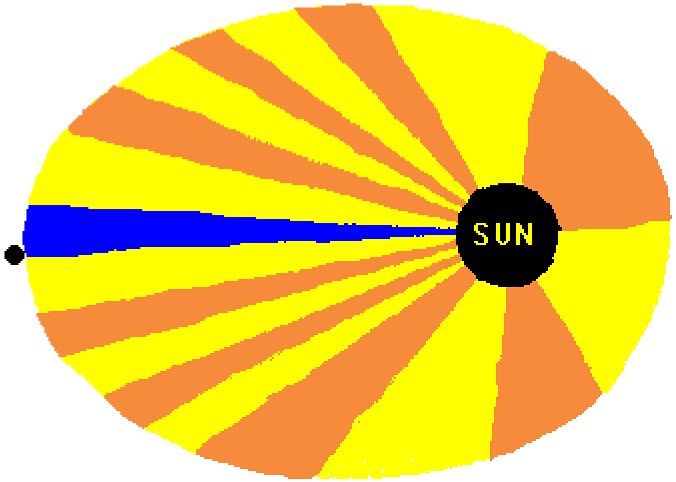
$$T^2 = \frac{4\pi(r_e + h)^3}{g_0}$$

$$T = \frac{2\pi}{\sqrt{g_o}}(r_e + h)^{3/2}$$

VELOCITY OF SATELLITE IN CIRCULAR ORBIT

If v_s is the velocity of the satellite, h is the height of the satellite and r_e is the radius of the earth then we can write:

$$v_s = \omega(r_e + h) = \frac{2\pi}{T}(r_e + h)$$



But

$$T = \frac{2\pi}{\sqrt{g_o}}(r_e + h)^{3/2}$$

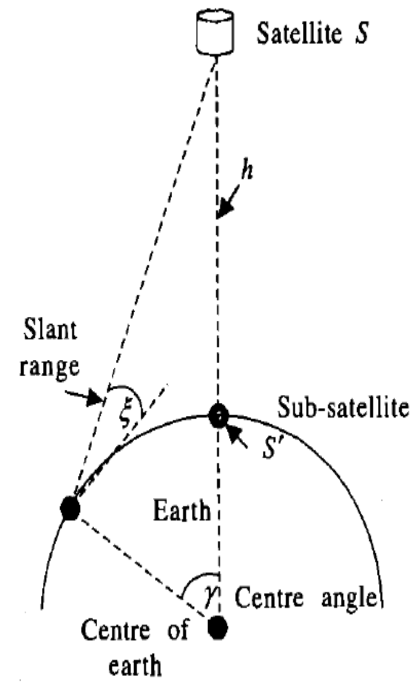
Therefore

$$v_s = \frac{(r_e+h)}{(r_e+h)^{3/2} / \sqrt{g_o}} = \sqrt{\frac{g_o}{(r_e+h)}}$$

This confirms Kepler's Second law that an increase in radius of the orbit leads to a decrease in the velocity of the satellite.

PARAMETERS OF GEOSYNCHNOUS SATELLITES

QUANTITY	EQUATION	AT SUB-SATELLITE POINT	AT GEOSTATIONARY ORBIT
Velocity	$v_s = \sqrt{\frac{g_o}{(r_e + h)}}$	7,905,364	3,074,689 km/s
Orbit Period	$T_s = \frac{2\pi d^{3/2}}{\sqrt{g_o}}$	5069.347	86,164.091 Sec
Angular Velocity	$\omega_s = \frac{\sqrt{g_o}}{d^{3/2}}$	0.001239	72.9211×10^{-6}rad/s



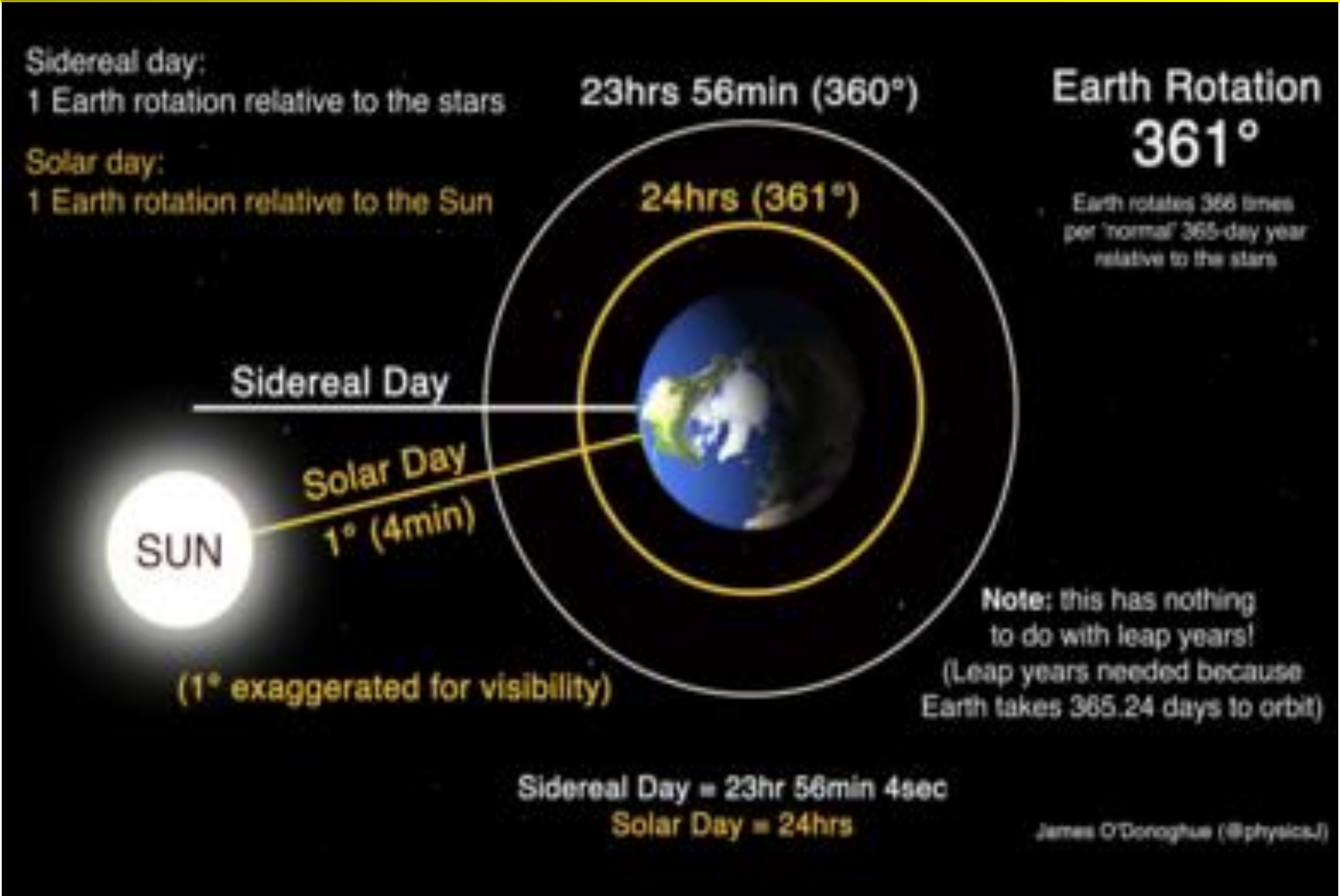
NOTE:

1. The Orbit period for a geostationary orbit is 86164.091/(60x60) = 24 hrs
2. On the other hand, if you were to keep an object flying at the sub-satellite point, then its orbital period will be 5069.347/(60x60) = 1.41 hrs.

SOLAR DAY AND SIDEREAL DAY / 01

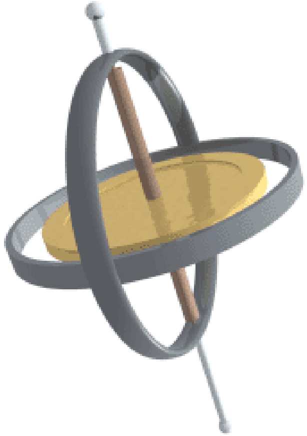
1. **Solar day** is the time it takes for the Earth to rotate about its axis so that the Sun appears in the same position in the sky (24 hrs).
2. Since both the sun and the earth are moving objects, a solar day of 24 hours does not give the accurate calculation.
3. For better accuracy a 'sidereal day' referred to fixed stars is used in satellite period calculations.
4. A **'sidereal day'** is 23 hours 56 minutes and 4 seconds or 0.9972 times the solar day.

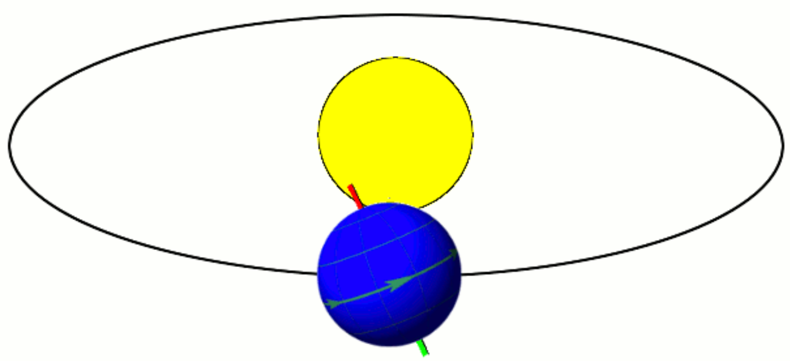
SOLAR DAY AND SIDEREAL DAY / 02



GYROSCOPIC STIFFNESS

- Just like natural satellites for an artificial satellite to consistently follow the same orbit in space, it must revolve round its pitch axis to create gyroscopic stiffness.





(a) Earth rotates round sun and also round its axis (asynchronous)

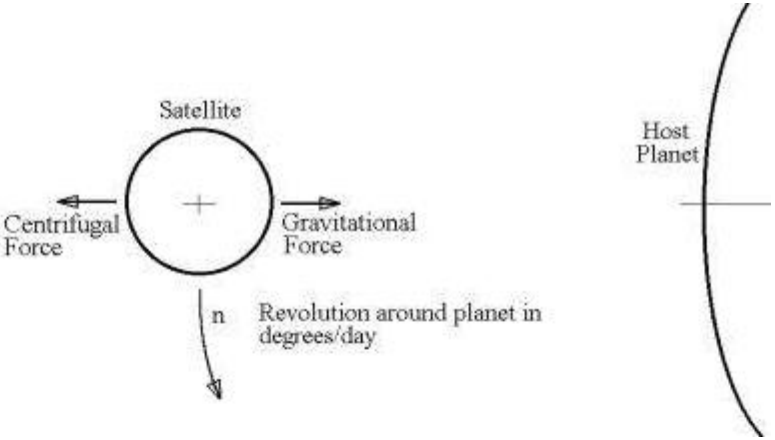
(b) The moon rotates round the earth and also round it polar axis (synchronous)

SATELLITE ORBITS – Departure from Keplarian Laws (1)

1. Keplerian orbit is ideal (not real) in the sense that it assumes the following:

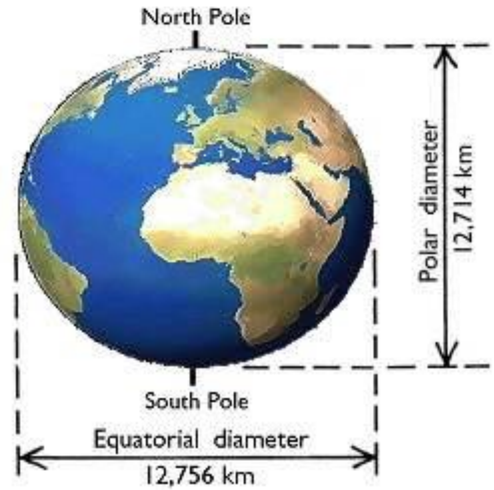
   (a) a uniform spherical earth surface

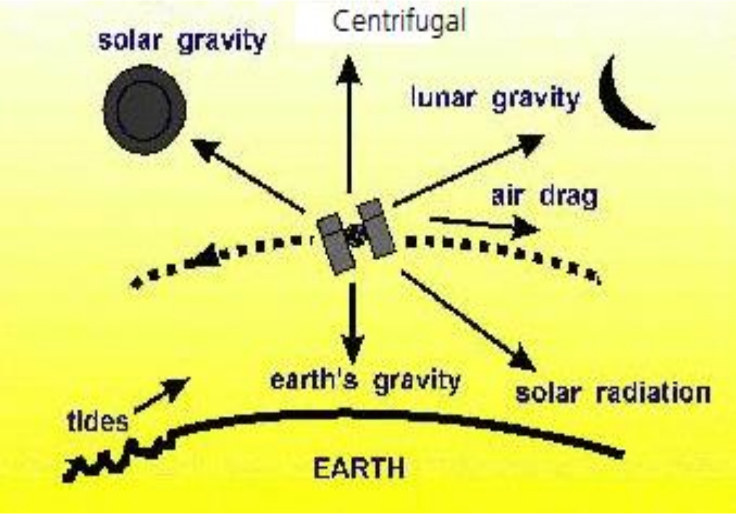
   (b) the only force acting on the satellite centrifugal.



SATELLITE ORBITS – Departure from Keplarian Laws (1)

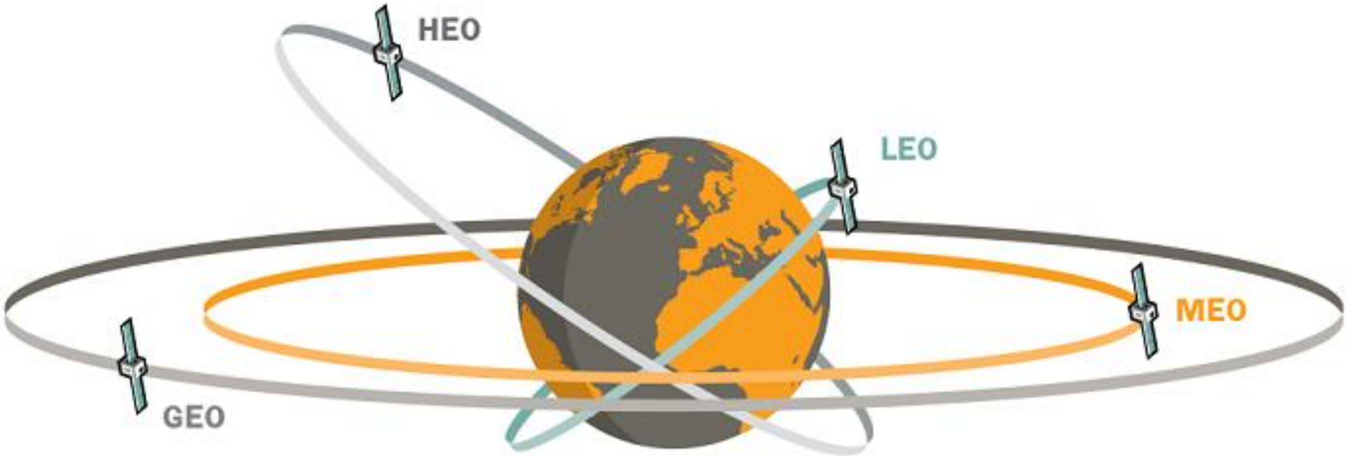
1. There are other external torques as follows:
   a) **Non-spherical earth:** The earth's polar diameter is 42 km shorter than the equatorial diameter.
   b) **Gravitational forces of other planetary bodies**, i.e sun, moon and others.
   c) **Atmospheric drag** (only in lower orbits)
   d) **Aerodynamic forces** (only in lower orbits)
   e) **Solar pressure/radiation**
   f) **Magnetic forces** acting on the satellite due to the earth's magnetic fields





GRAVITATIONAL PULL AND ATMOSPHERIC DRAG

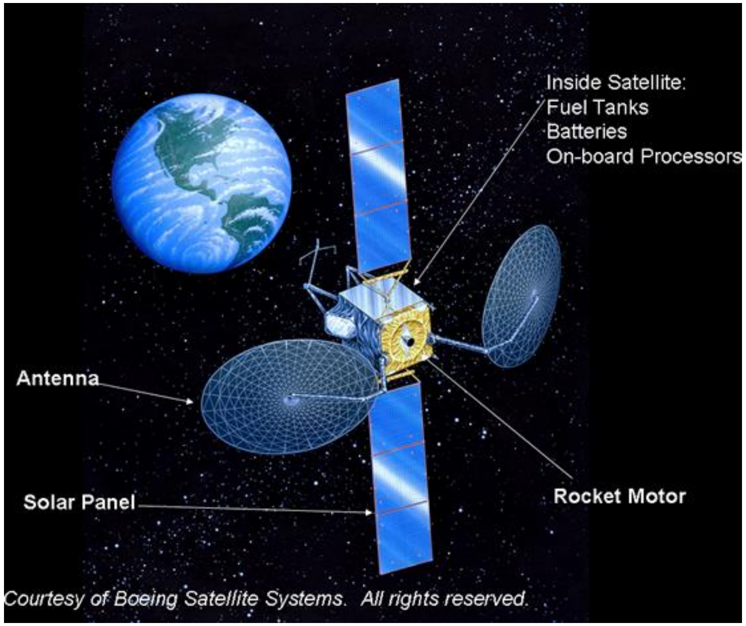
1. Gravitational pull of the moon and sun have little effect on satellites in Low Earth Orbits (LEO) but the have effect on Geostationary Orbit satellites.
2. Atmospheric drag has little effect on geostationary satellites but have effect on LEO satellites especially below 1,000 kms.



Orbit Distance	Km	1-way Delay
Low Earth Orbit (LEO)	160 - 1,400	50 ms
Medium Earth Orbit (MEO)	10,000 - 25,000	100 ms
Geostationary Earth Orbit (GEO)	36,000	250 ms

SOLAR PRESSURE

1. Sun rays, which are essentially photons, exert a typical for of 4.63×10^{-6} Newtons/m^2 when they fall on a satellite.
2. This force is sufficient to tilt the satellite in space over a long period of time.



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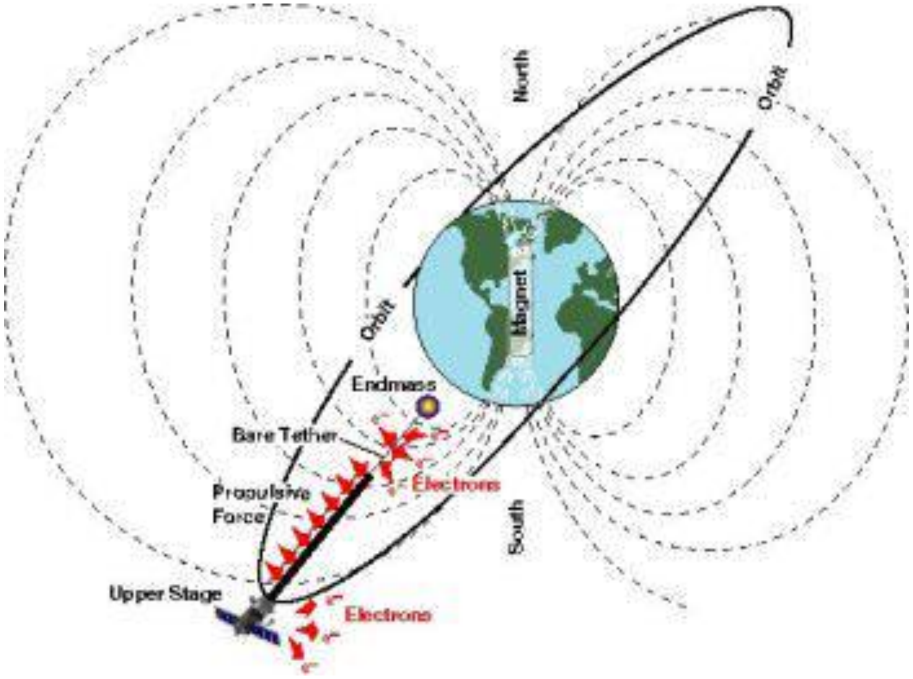
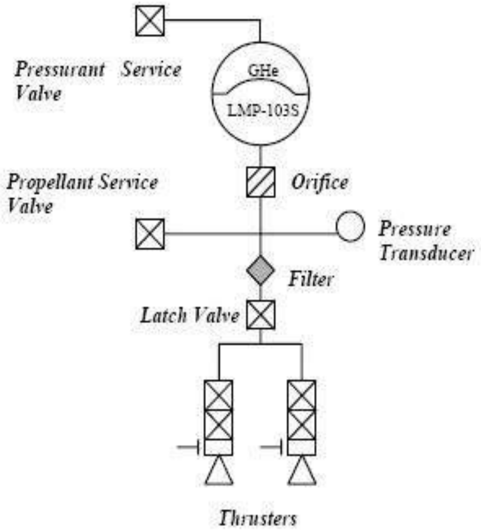
EARTH'S MAGNETIC FIELD

- The earth's magnetic field is significant in LEO Satellites but negligible influence on satellites in geostationary orbits.



EFFECT OF SATELLITE INTERNAL TORGUE

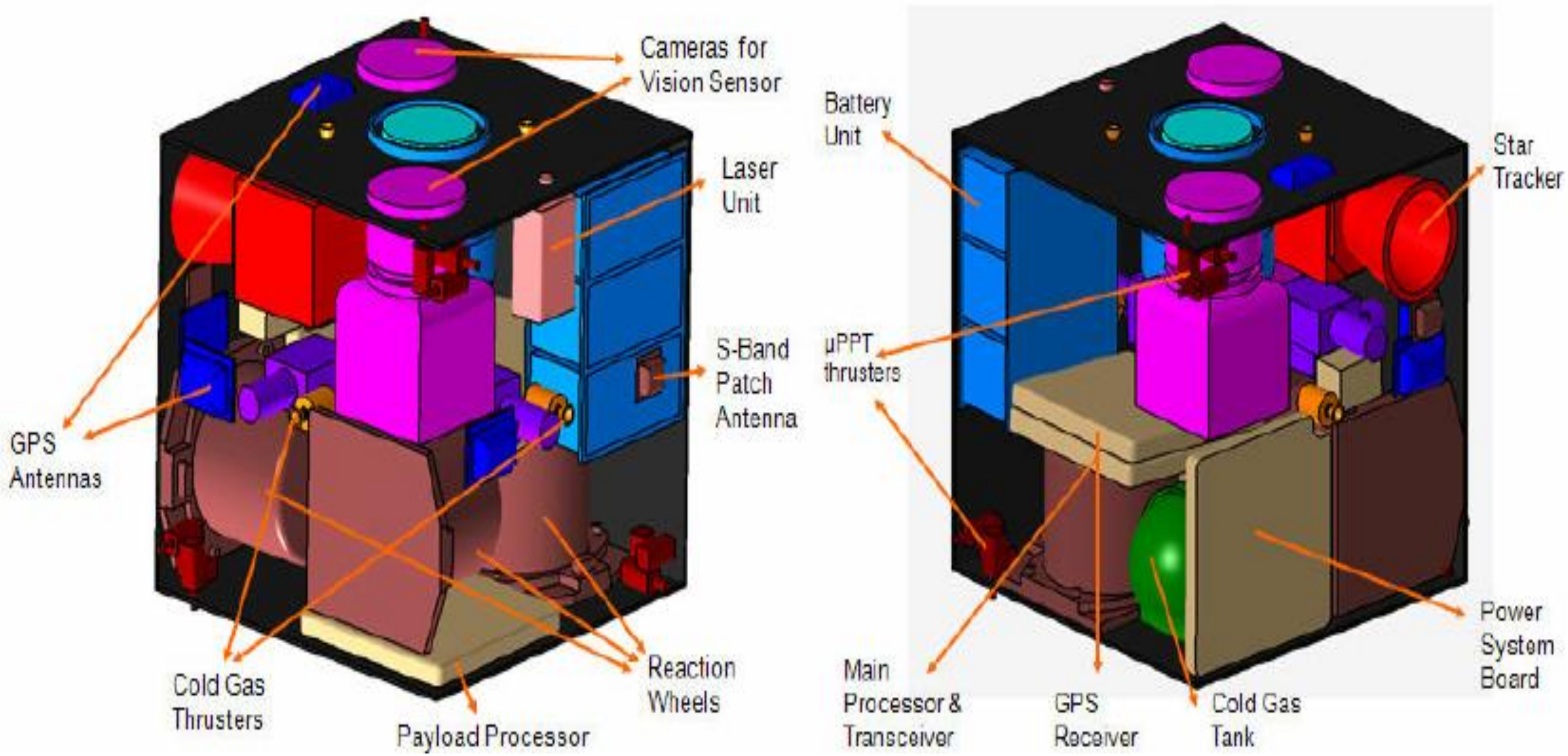
- The following internal torque influence the satellite attitude:
  a) Thruster forces can change the stability of the satellite
  b) Fuel movement (usually liquid hydrazine) in the satellite can change the attitude and orbital position of a satellite.



Schematic view of hydraulics system

Layout of propulsion system

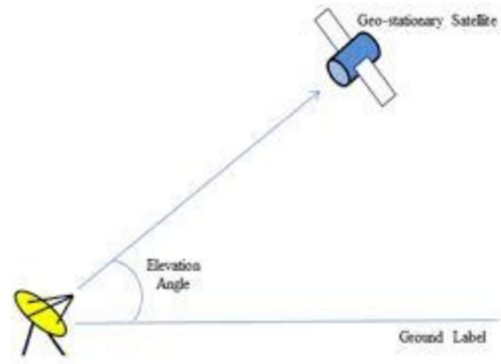
TYPICAL SMALL SATELLITE LAYOUT



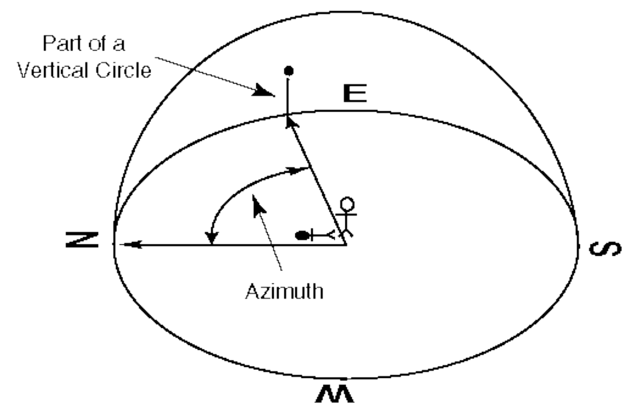
SATELLITE POSITION ABOVE THE EARTH

- The position and parking place (orbital slot) of a satellite can be determined by the following parameters:
  a) The elevation angle

  b) The azimuth angle

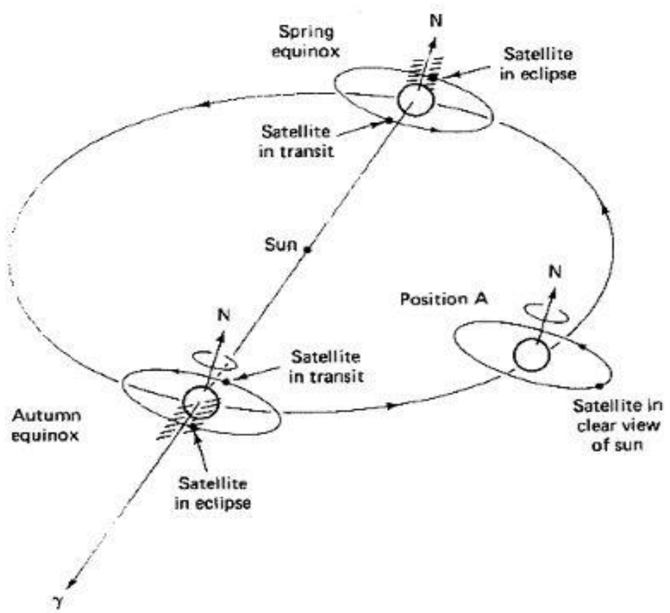


Azimuth = 80°



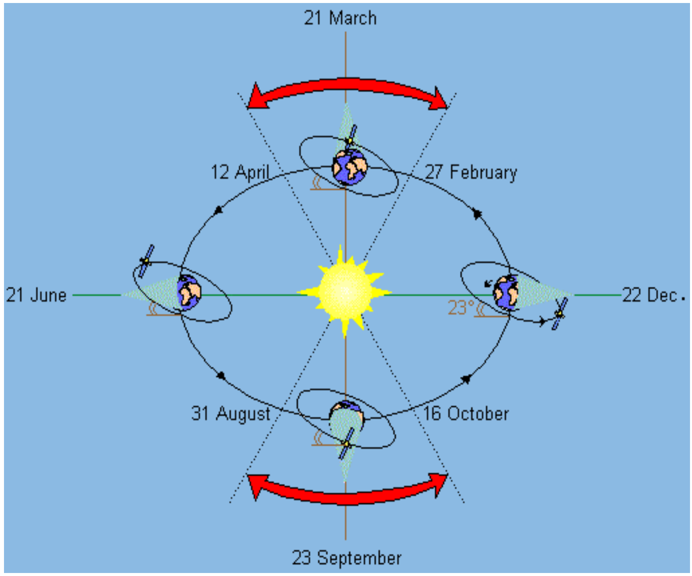
SATELLITE ECLIPSE

- A **satellite** is said to be in eclipse when the earth or moon prevents sunlight from reaching it.
- **Solar eclipses** are important as they affect the working of the satellite because during eclipse satellite receives no power from its solar panels and it has to operate on its onboard standby batteries which reduce satellite life.



EFFECT OF SOLAR ECLIPSE ON SATELLITE PERFORMANCE

1. Most satellites go through a period when the solar radiation necessary for powering the system is shadowed by earth. This period is called the eclipse of the satellite.
2. The period of eclipse varies from satellite to satellite depending on the type of orbit and orbital path.
3. In geostationary satellites, the period falls during the equinox period.
4. Equinox period in around 21st March and 23rd September when night and day are equal (*hence the name equinox*).



Satellite most affected by eclipse phenomena between:

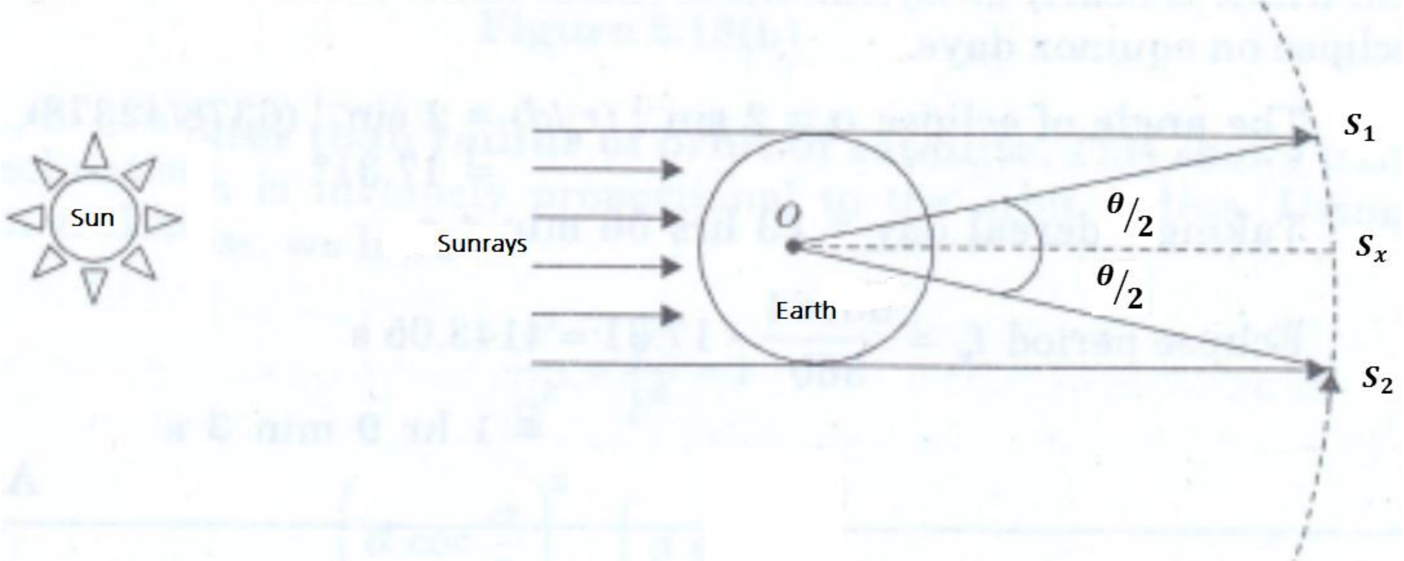
27 February to 12 April, and

31 August to 16 October

CALCULATING THE DURATION A SATELLITE IS UNDER ECLIPSE

- **Assumptions:**

1. Rays are parallel beams of photons and are not affected by the earth's environment.
2. The eclipse occurs only on equinox day when the satellite is completely eclipsed
3. The eclipse starts at S_1 and ends at S_2 as shown.



CALCULATING THE DURATION A SATELLITE MAY BE UNDER ECLIPSE

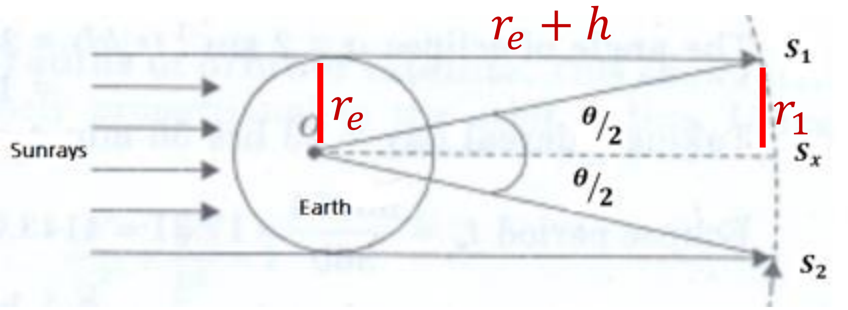
1. Since the radius of the orbit is very large, $r_1 = S_xS_1$ can be considered as a straight line.

$$\frac{r_e}{r_e + h} = \sin\left(\frac{\theta}{2}\right)$$

Hence

$$\theta = 2sin^{-1}\left(\frac{r_1}{r_e + h}\right)$$

Where r_e is the radius of the earth and h is the height of the satellite.



$$r_e = r_1 \approx S_1S_x$$

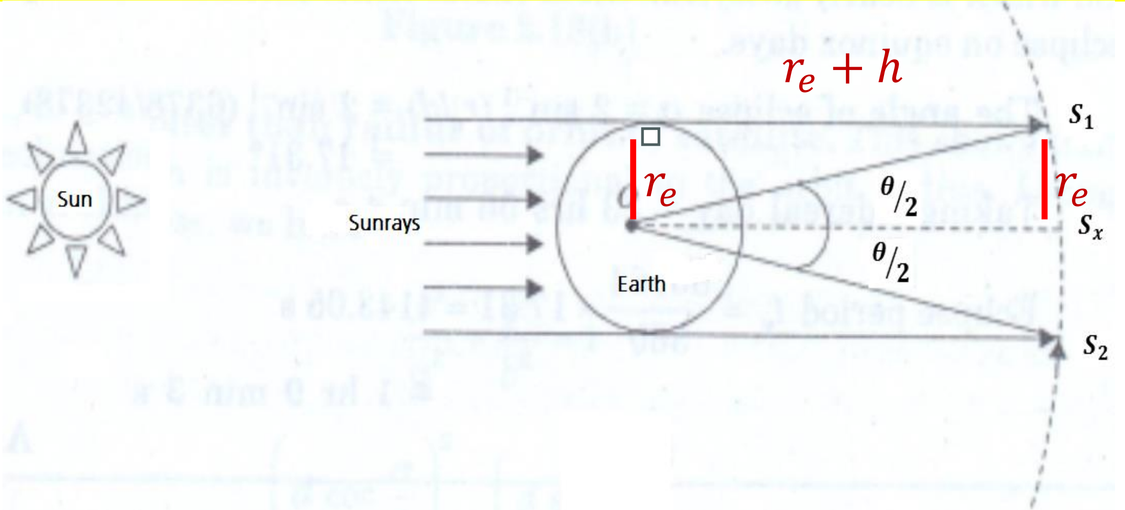
2. For a geostationary satellite the duration of the eclipse t_e can be estimated as:

$$t_e = \frac{\theta}{360} Orbital\ period$$

EXAMPLE: ECLIPSE PERIOD

- Consider a satellite at a height 36,000 km and orbiting the equatorial plane.
- Establish if the satellite will be under ecllipse on equinox days and if so, the duration of the eclipse.
- Take the radius of the earth as 6,378 km

SOLUTION



- Height, h = 36,000
- Radius of the earth = 6378 km
- Radius of the orbit is 36,000+6,378 = 42,378
- Angle of eclipse θ is given by

$$\theta = 2sin^{-1}\left(\frac{r_e}{d}\right) = 2sin^{-1}\left(\frac{6378}{42378}\right) = 17.31^o$$

- Sidereal day = 23hrs 56 min 4 seconds
- Eclipse period $=\frac{17.31}{360} \times (4 + 56 \times 60 + 23 \times 3600)$

$$= \frac{17.31}{360} \times 86164 = 4{,}143 \ \ Sec$$

= 1 hr 9 minutes 3 sec

TAKE AWAY EXAMPLE

- The orbital period of a satellite is 650 min. Determine the semi-major axis of the elliptical orbit.

- Take $\mu = GM = 6.67 \text{ X } 10^{-l1} \text{ X } 5.98 \text{ X } 10^{24}$