



ECE 514E – RADAR & SATELLITE ENGINEERING

RADAR EQUATION – STUDY GUIDE/REVISION

1. INTRODUCTION TO THE RADAR EQUATION

Purpose

The radar equation quantifies the power received by a radar system from a target. It is fundamental for designing radar systems, predicting performance, and analysing factors like range, sensitivity, and target detection.

Key Applications

- Military (target tracking)
- Aviation (air traffic control)
- Weather monitoring (precipitation detection)
- Automotive (collision avoidance)

2. DERIVATION OF THE BASIC RADAR EQUATION

The radar equation models the relationship between transmitted power, target properties, and received power.

Step-by-Step Derivation:

1. Power Density at Target:

Transmitted power (P_t) spreads over a sphere. At range R , power density is:

$$S_{inc} = \frac{P_t G_t}{4\pi R^2} \quad S_{inc} = \frac{4\pi R^2 P_t G_t}{4\pi R^2}$$

- G_t : Transmit antenna gain.

2. Power Intercepted by Target:

Target reflects power based on radar cross-section (σ , in m^2):

$$P_{reflected} = S_{inc} \cdot \sigma = \frac{P_t G_t \sigma}{4\pi R^2} \quad P_{reflected} = \frac{S_{inc} \cdot \sigma}{4\pi R^2} = \frac{4\pi R^2 P_t G_t \sigma}{4\pi R^2}$$

3. Power Density at Receiver:

Reflected power spreads back to the radar. Power density at the receiver antenna:

$$S_{rec} = \frac{P_{reflected}}{4\pi R^2} = \frac{P_t G_t \sigma}{(4\pi)^2 R^4} \quad S_{rec} = \frac{4\pi R^2 P_{reflected}}{(4\pi)^2 R^4} = \frac{4\pi R^2 P_t G_t \sigma}{(4\pi)^2 R^4}$$

4. Received Power (P_r):

Effective aperture area of the receive antenna (A_e) captures power:

$$P_r = S_{rec} \cdot A_e = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4} \quad P_r = S_{rec} \cdot A_e = \frac{(4\pi)^3 R^4 P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4}$$

- G_r : Receive antenna gain (often $G_t = G_r = G$ for monostatic radar).

- λ : Wavelength ($\lambda=c/f$, c = speed of light, f = frequency).
- **Note:** $A_e = G \lambda^2 / 4\pi$ (antenna theory).

3. KEY PARAMETERS & DEFINITIONS

Parameter	Symbol	Unit	Description
Transmitted Power	P_t	W	Peak power emitted by radar.
Antenna Gain	G_t, G_r	Dimensionless	Directional efficiency relative to isotropic radiator.
Radar Cross-Section	σ	m^2	Measure of target's reflectivity.
Wavelength	λ	m	c/f ($c=3 \times 10^8$ m/s).
Range	R	m	Distance to target.
Received Power	P_r	W	Power returned to radar.

4. PRACTICAL MODIFICATIONS & LOSSES

Real-world radars include losses and noise:

- **System Losses (LL):**

$$P_r = P_t G^2 \lambda^2 \sigma / (4\pi)^3 R^4 L \quad (L \geq 1) \quad P_r = (4\pi)^3 R^4 L P_t G^2 \lambda^2 \sigma / (L \geq 1)$$

Accounts for transmission line loss, atmospheric attenuation, etc.

- **Minimum Detectable Signal (S_{min}):**

Smallest P_r the radar can detect:

$$R_{max} = P_t G^2 \lambda^2 \sigma / (4\pi)^3 S_{min} L \quad R_{max} = 4(4\pi)^3 S_{min} L P_t G^2 \lambda^2 \sigma$$

- S_{min} depends on receiver noise temperature (T), bandwidth (B), and noise figure (F).

- **Signal-to-Noise Ratio (SNR):**

$$SNR = P_r / kTB \quad SNR = kTB P_r$$

- k : Boltzmann's constant (1.38×10^{-23} J/K).

5. EXAMPLE PROBLEMS

Problem 1: Calculate P_r for a radar with:

$$P_t = 1 \text{ MW}, G = 35 \text{ dB}, \lambda = 0.03 \text{ m}, \sigma = 1 \text{ m}^2, R = 100 \text{ km}, L = 2$$

Solution:

- Convert G from dB: $G = 10^{35/10} = 3162$
- Use radar equation:

$$P_r = (106) \cdot (3162)^2 \cdot (0.03)^2 \cdot 1 \cdot (4\pi)^3 \cdot (100,000)^4 \cdot 2 \approx 1.11 \times 10^{-12} \text{ W} \cdot P_r$$

$$= (4\pi)^3 \cdot (100,000)^4 \cdot 2 \cdot (106) \cdot (3162)^2 \cdot (0.03)^2 \cdot 1 \approx 1.11 \times 10^{-12} \text{ W}.$$

Problem 2: Find R_{\max} if $S_{\min} = 10^{-13} \text{ W}$ (other parameters as above).

Solution:

$$R_{\max} = 106 \cdot (3162)^2 \cdot (0.03)^2 \cdot 1 \cdot (4\pi)^3 \cdot 10^{-13} \cdot 2 \approx 141 \text{ km}.$$

$$R_{\max} = 4 \cdot (4\pi)^3 \cdot 10^{-13} \cdot 2 \cdot 106 \cdot (3162)^2 \cdot (0.03)^2 \cdot 1 \approx 141 \text{ km}.$$

6. PRACTICE PROBLEMS

1. A radar operates at 10 GHz with $P_t = 100 \text{ kW}$, $G = 30 \text{ dB}$, $\sigma = 5 \text{ m}^2$, $R = 50 \text{ km}$, and $L = 3$. Find P_r .
2. Derive R_{\max} if $P_r = S_{\min} = 5 \times 10^{-14} \text{ W}$ for the radar in Problem 1.
3. How does P_r change if R doubles? If σ increases by 6 dB?

7. COMMON PITFALLS & TIPS

- **Units:** Ensure consistency (e.g., R in meters, not km).
- **Decibels:** Convert gains/losses from dB to linear scale: $G_{\text{linear}} = 10^{G_{\text{dB}}/10}$.
- **Range Dependence:** $P_r \propto 1/R^4$ → Doubling range reduces power by 16×!
- **σ Variability:** Highly dependent on target shape, material, and orientation.

8. ADVANCED EXTENSIONS

- **Bistatic Radar:** Separate transmit/receive antennas ($R_t \neq R_r$).
- **Radar Clutter:** Unwanted echoes from ground/weather.
- **Pulse Integration:** Combining multiple pulses to improve SNR.